

Homework: 2.2 / 31-40

35) $f(x) = \frac{x^3 - 1}{x - 1}$ substituting $\Rightarrow \frac{0}{0}$ [DK]

a) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x^2+x+1)}{x-1}$

$= \lim_{x \rightarrow 1} (x^2 + x + 1) = 3$

b) sketch graph:



graph $(x^2 + x + 1)$

graph $(x^2 + x)$
 $x(x+1)$

$\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) + 1$

$\frac{1}{4} - \frac{1}{2} + 1 = \frac{1}{4} - \frac{2}{4} + \frac{4}{4}$

36
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39-40

$$36) f(x) = \begin{cases} \frac{x^2-9}{x+3}, & x \neq -3 \\ \text{?} -6, & x = -3 \end{cases} = (x-3)$$

a) find $k \ni k = f(-3) = \lim_{x \rightarrow -3} f(x) = -6$

$$\lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} \frac{x^2-9}{x+3} = \lim_{x \rightarrow -3} \frac{(x-3)(x+3)}{x+3}$$

$$= \lim_{x \rightarrow -3} (x-3) = -6$$

$$k = -6$$

$$37) \quad (*) \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{x^2} \right) = \lim_{x \rightarrow 0^+} \frac{1}{x} - \lim_{x \rightarrow 0^+} \frac{1}{x^2} = +\infty - (+\infty) = 0$$

a) why is this "incorrect"

b) show $\star = -\infty$

$$\lim_{x \rightarrow 0^+} \frac{x-1}{x^2}$$

-
+

→ DNE
 → $+\infty$
 → $-\infty$ (circled in red)

$$39) \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} \quad (\text{yada yada IDK})$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} \cdot \left(\frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(x+4) - 4}{x(\sqrt{x+4} + 2)} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+4} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+4} + 2} = \frac{1}{2+2} = \frac{1}{4}$$

$$40) \lim_{x \rightarrow 0} \frac{\sqrt{x^2+4} - 2}{x}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sqrt{x^2+4} - 2}{x} \right) \left(\frac{\sqrt{x^2+4} + 2}{\sqrt{x^2+4} + 2} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(x^2+4) - 4}{x(\sqrt{x^2+4} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{x^2}{x(\sqrt{x^2+4} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{x}{\sqrt{x^2+4} + 2} = \frac{0}{4} = 0$$

$$36) \quad f(x) = \begin{cases} \frac{x^2-9}{x+3}, & x \neq -3 \\ k, & x = -3 \end{cases} = \underline{x-3}$$

a) find $k \ni f(-3) = \lim_{x \rightarrow -3} f(x) = -6$

Polly Normal:

$$C_n x^n + C_{n-1} x^{n-1} + \dots + C_1 x + C_0$$

$$\lim_{x \rightarrow -3} \frac{x^2-9}{x+3} \xrightarrow{0/0} \text{DK}$$

$$= \lim_{x \rightarrow -3} \frac{(x-3)(x+3)}{(x+3)} = \lim_{x \rightarrow -3} (x-3) = -6$$

$$\boxed{k = -6}$$

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$$\underline{37)} \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{x^2} \right) = \lim_{x \rightarrow 0^+} \frac{1}{x} - \lim_{x \rightarrow 0^+} \frac{1}{x^2} = (+\infty) - (+\infty) = 0$$

a) why am I stupid?

b) $\lim_{x \rightarrow 0^+} \frac{x-1}{x^2}$

Diagram illustrating the limit process:

- Numerator: $x-1$ (with a green minus sign above it) approaches -1 (indicated by a red arrow).
- Denominator: x^2 (with a green plus sign below it) approaches 0 (indicated by a red arrow).
- The resulting form is $\frac{-1}{0}$.
- From $\frac{-1}{0}$, three paths are shown with green lines:
 - Upwards to $-\infty$ (which is circled in green).
 - Downwards to $+\infty$.
 - Outwards to DNE (Does Not Exist).

Remember
 $\lim_{x \rightarrow 0} \frac{1}{x}$

$$39) \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} \rightarrow \frac{0}{0} \text{ IDK}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sqrt{x+4} - 2}{x} \right) \left(\frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(x+4) - 4}{x(\sqrt{x+4} + 2)} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+4} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{(\sqrt{x+4} + 2)} = \frac{1}{2+2} = \frac{1}{4}$$

$$40) \lim_{x \rightarrow 0} \frac{\sqrt{x^2+4} - 2}{x}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sqrt{x^2+4} - 2}{x} \right) \left(\frac{\sqrt{x^2+4} + 2}{\sqrt{x^2+4} + 2} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(x^2+4) - 4}{x(\sqrt{x^2+4} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{x^2}{x(\sqrt{x^2+4} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{x}{\sqrt{x^2+4} + 2}$$

$$= \frac{0}{4} = 0$$

35) $f(x) = \frac{x^3 - 1}{x - 1}$

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x^2+x+1)}{(x-1)}$$

$$= \lim_{x \rightarrow 1} x^2 + x + 1 = 1 + 1 + 1 = 3$$

b) graph $\frac{x^3 - 1}{x - 1}$ \nearrow $x^2 + x + 1, x \neq 1$

